ME 340.343 Design and Analysis of Dynamic System.

Lecture 23, More Control examples and Stability

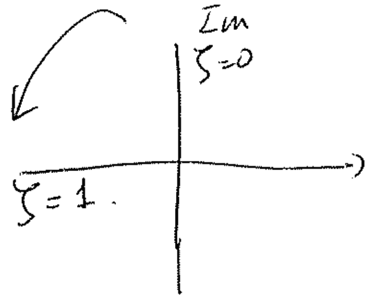
Today's Objectives,

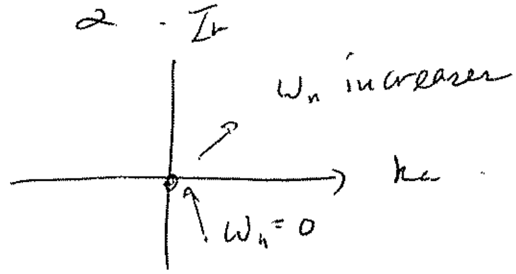
- s-plane controller design.

Remember for linear 2nd order equation

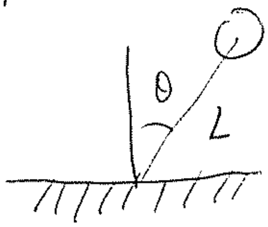
$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 0$$

The poles are $\quad p = \dfrac{-2\omega_n\zeta \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2}$





S-plane Controller Design. (Inverted Pendulum Example)



$$m\ddot{x} + b\dot{x} - kx = f(t)$$ ← control signal

$k>0$.

$$p = \frac{-\frac{b}{m} \pm \sqrt{\left(\frac{b}{m}\right)^2 + 4\frac{k}{m}}}{2}.$$

Stable if $\quad -\frac{b}{m} + \sqrt{\left(\frac{b}{m}\right)^2 + \frac{4k}{m}} < 0.$

Derivative Control

$$m\ddot{x} + b\dot{x} - kx = f(t) = c_1 \dot{x}$$

$$p = \frac{-\left(\frac{b}{m} - \frac{c_1}{m}\right) \pm \sqrt{\left(\frac{b}{m} - \frac{c_1}{m}\right)^2 + \frac{4k}{m}}}{2}$$

Doesn't really help stability.

Proportional Control

$$m\ddot{x} + b\dot{x} - kx = f(t) = c_2 x$$

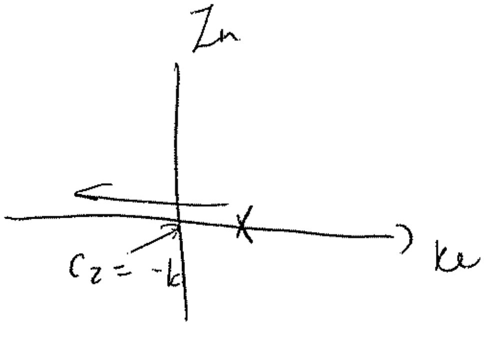
$$p = \frac{-\frac{b}{m} \pm \sqrt{\left(\frac{b}{m}\right)^2 + \frac{4(k+c)}{m}}}{2}$$

$$-\frac{b}{m} + \sqrt{\left(\frac{b}{m}\right)^2 + \frac{4}{m}(k+c_2)} < 0$$

$$\left(\frac{b}{m}\right)^2 + \frac{4}{m}\left(\frac{k+c_2}{n}\right) < \left(\frac{b}{m}\right)^2$$

$$k + c_2 < 0$$

$$c_2 < -k$$



Suppose $\left(\frac{b}{m}\right)^2 + \frac{4}{m}(k+c_2) = 0$.

$$P_1 = \frac{-\left(\frac{b}{m}\right) \pm 0}{2} \qquad P_1 = P_2 = -\frac{b}{2m}.$$

~~Deg~~ Degenerate poles! (Critical damping).

Both Derivative and Proportional Control

$$f(t) = c_1 \dot{x} + c_2 x$$

$$p = \frac{-\left(\frac{b}{m} - \frac{c_1}{m}\right) \pm \sqrt{\left(\frac{b}{m} - \frac{c_1}{m}\right)^2 + \frac{4(k+c_2)}{m}}}{2}.$$

You can ~~get any pole~~ move the poles to any location on the complex plane.

Open-loop Control   Unstable to Stable.

$$m\ddot{x} - kx = (-a\cos\gamma t)\,x$$

moving end of the pendulum, vertical oscillation

$$m\ddot{x} - (k - a\cos\gamma t)\,x = 0.$$

if $\gamma \gg \sqrt{\frac{k}{m}}$, and a is small, then there is a rapid

oscillation on top of a smooth motion.

$$x = X(t) + \delta(t)$$

$$m\ddot{X} - \underbrace{\left(k - \frac{a^2}{4m\gamma^2}\right)}_{k_{eff}} X = 0.$$

k_{eff} can be positive! Stable motion!