530.327 FALL 08 – HW8 SOLUTIONS

PROBLEM: NEW 'VISCOUS' FORCE → SHEAR STRESS $\tau_{21} = \chi u_1$
→ NORMAL STRESS $\tau_{11} = \chi u_1$ (SAME NOTATION AS HANDOUT)

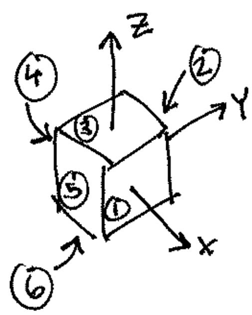
- TO DERIVE THE APPROPRIATE FORM OF THE N-S EQUATIONS:

! NOTE: IN MOMENTUM EQ., ALMOST EVERYTHING IS UNCHANGED

$$\underbrace{\vec{F}_{S,P}}_{\text{(SURFACE PRESSURE FORCES) UNCHANGED}} + \underbrace{\vec{F}_{S,V}}_{\text{(VISCOUS SURFACE FORCE) CHANGED}} + \underbrace{\vec{F}_B}_{\text{(BODY FORCE) UNCHANGED}} = \underbrace{\frac{\partial}{\partial t}\int_{CV} \rho \vec{V}\, d\forall}_{\text{UNCHANGED}} + \underbrace{\int_{CS} \rho \vec{V}(\vec{V}\cdot d\vec{A})}_{\text{UNCHANGED}}$$

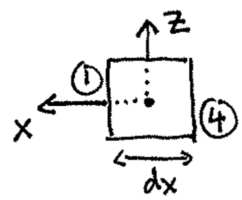
3 PTS

- FOR THE C.V. FROM THE NOTES →



FOCUSING ON THE $\boxed{\text{z - COMPONENT}}$ →

- FACES 1 & 4



2 PTS

(1): FORCE $= \chi\Big(\underbrace{w + \frac{\partial w}{\partial x}\cdot\frac{dx}{2}}_{\text{W AT } x=\frac{dx}{2}\text{ (TAYLOR EXPANSION)}}\Big)\underbrace{dy\,dz}_{\text{AREA}}$

(4): FORCE $= -\chi\left(w - \frac{\partial w}{\partial x}\frac{dx}{2}\right)dy\,dz$

2 PTS

SUM OF FORCES ON (1) AND (4): $\boxed{= \chi\frac{\partial w}{\partial x}dx\,dy\,dz}$ (*)

PROBLEM 1 - p.2

- FACES 2&5 AND 3&6 ARE EXACTLY ANALOGOUS TO FACES 1&4

$$\text{FORCES} = \boxed{\chi \frac{\partial w}{\partial y} dx\,dy\,dz} \quad (**) \qquad \text{FORCES} = \boxed{\chi \frac{\partial w}{\partial z} dx\,dy\,dz} \quad (***)$$

1 PT

∴ TOTAL VISCOUS FORCES IN Z-DIRECTION (* + ** + ***)

$$= \chi \left(\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} \right) dx\,dy\,dz \quad (****)$$

3 PTS

- THE ORIGINAL Z-COMPONENT OF THE N-S EQS WAS (EQ. 21 IN THE NOTES)

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -g - \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

FROM (****):

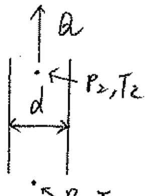
$\mu(\cdots)$ GETS REPLACED BY $\chi\left(\frac{\partial w}{\partial x} + \cdots\right)$

∴ 'NEW' Z-COMPONENT OF THE N-S EQUATIONS IS

$$\boxed{\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -g - \frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\chi}{\rho}\left(\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}\right)}$$

4 PTS

Problem 2



$d = 4.09$ cm.

$Q = 200\ \ell/min = \frac{200}{1000 \times 60}\ \frac{m^3}{s} = \frac{1}{300}\ m^3/s$

$\rho = 999\ kg/m^3$

$C_v = 4.186\ J/g\cdot°C = 4186\ J/kg°C$

(a)

$\Delta P = P_1 - P_2 = 1.25 \times 10^4\ N/m^2$

$\Delta T = T_1 - T_2 = ?$

from the conservation of energy.

$$\dot{Q} - \dot{W} = \frac{dE}{dt} = 0. = \frac{\partial}{\partial t}\int_{CV} \cancel{e}\, dm + \int_{CS} (e + PV)\,\rho\ \vec{V}\cdot d\vec{A}$$

$$u_1 + \frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = u_2 + \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

$$\Rightarrow (u_2 - u_1) + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) + \frac{1}{\rho}(P_2 - P_1) = 0 \quad 2$$

$$\Rightarrow C_v \underbrace{(T_2 - T_1)}_{= -\Delta T} + g(z_2 - z_1) - \frac{1}{\rho}\Delta P = 0$$

$$C_v \Delta T = g(z_2 - z_1) - \frac{1}{\rho}\Delta P$$

$$\Delta T = \frac{1}{C_v}\left(g(z_2 - z_1) - \frac{1}{\rho}\Delta P\right) = \frac{1}{4186}\left(9.81 \times 1 - \frac{1}{999} \times 1.25 \times 10^4\right)$$

$$= -0.00065\ °C \quad \#$$

3 take 1 pt for ~~nega~~ positive sign

(b)

η = pump efficiency = $\frac{W_{out}}{W_{in}}$

$$\dot{Q} - \dot{W} = \frac{\partial}{\partial t}\int_{CV} e\rho\, dV + \int_{CS} \left(e + \frac{P}{\rho}\right) \rho\ \vec{V}\cdot d\vec{A},$$

$\dot{W}$ = work system done on environment

= negative work environment done on system.

$= -W_{out} = -\eta W_{in}$

(b) (continue)

$$\dot{Q} - \dot{W} = \eta W_{in} = \left[-\left(u_1 + \frac{V_1^2}{2} + \frac{P_1}{\rho} + g z_1\right) + \left(u_2 + \frac{V_2^2}{2} + \frac{P_2}{\rho} + g z_2\right)\right] Q\rho \quad ②$$

$$W_{in} = \frac{1}{\eta}\left(C_v(T_2 - T_1) + \frac{1}{\rho}(P_2 - P_1) + g(z_2 - z_1)\right) Q\rho$$

$$= \frac{1}{0.3}\left(4186 \times 0.0008 - \frac{1}{999}\ 9790 + 9.81\right)\frac{1}{300} \times 999$$

$$= 37.28 \ \text{J/s} \ \#$$ ③ , take ① pt for negative sign

Problem 3. $d = 1.049/12$ ft

a.

$$Q = \frac{\pi d^2}{4} V$$

$$\Rightarrow V = \frac{4Q}{\pi d^2} = \frac{4 \times 0.5 \ \text{gal/min}}{\pi \times (1.049)^2 \ \text{in}^2} = \frac{4 \times \frac{0.5}{7.481 \times 60} \ \text{ft}^3/\text{sec}}{\pi \times \left(\frac{1.049}{12}\right)^2 \ \text{ft}^2} = 0.186 \ \text{ft/sec}$$

$\rho = 1.93$ slug/ft^3, $\mu = 2.05 \times 10^{-5}$ lbf·s/ft^2 ②

$$Re = \frac{\rho V D}{\mu} = \frac{1.93 \times 0.186 \times 1.049/12}{2.05 \times 10^{-5}} = 1531 < 2300$$

so it is laminar # ③

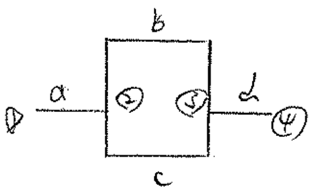
b.

$$h_l = \frac{\Delta P}{\rho} = \frac{P_1 - P_4}{\rho}, \quad P_4 = 0, \quad L = L_{in} + L_{out} + L_e = 5 + 5 + 15 = 25$$

$$\Rightarrow P_1 = \rho h_l = \rho \frac{64}{Re}\frac{L}{D}\frac{V^2}{2} = 1.93 \ \frac{64}{1531} \ \frac{25}{\frac{1.049}{12}} \ \frac{0.186^2}{2} = 0.399 \ \text{lbf/ft}^2 \ \#$$ ⑤

(2117.2)
Pabs 14.73 psi

c.



$Q_b + Q_c = Q_a$ ②

$h_l b = h_l c$, $\quad h_l = \frac{64}{Re}\frac{L}{D}\frac{V^2}{2} = \frac{64\mu}{\rho V D}\frac{L}{D}\frac{V^2}{2} = 32\frac{\mu L V}{\rho D^2}$

$Q = VA = V\frac{\pi d^2}{4}$, $\Rightarrow h_l = \frac{128 \mu L Q}{\pi \rho D^4}$

$$\Rightarrow \frac{128 \mu L_e Q_c}{\pi \rho D^4} = \frac{128 \mu L_t Q_b}{\pi \rho D^4} \Rightarrow \frac{Q_c}{Q_b} = \frac{L_t}{L_l} = \frac{30}{15} = 2$$

$$\Rightarrow Q_b = \frac{1}{3} Q_a = \frac{1}{3} \times 0.5 = \frac{1}{6} = 0.167 \ \text{gal/min} \ \#$$ ③ $= 3.72 \times 10^{-4}$ ft^3/s