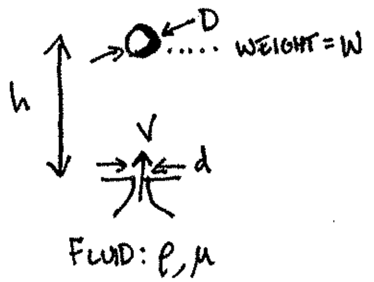
530.327 FALL 08
HW 6 SOLUTIONS

PROBLEM 1



KNOW: $h = fn(D, d, V, \rho, \mu, W)$

$h \to [L]$; $D \to [L]$; $d \to [L]$; $V \to \left[\frac{L}{T}\right]$; $\rho \to \left[\frac{M}{L^3}\right]$; $\mu \to \left[\frac{M}{LT}\right]$; $W \to \left[\frac{ML}{T^2}\right]$ (1)

- $n = 7$; $m = 3$; $n - m = 4$ DIMENSIONLESS PARAMETERS (1)

- USING $\boxed{d, V, \rho}$ AS REPEATING PARAMETERS

$\Pi_1 = h\, d^a V^b \rho^c \Rightarrow \boxed{\Pi_1 = \frac{h}{d}}$ BY INSPECTION

$\boxed{\Pi_2 = \frac{D}{d}}$ BY INSPECTION

$\Pi_3 = \mu\, d^a V^b \rho^c \rightarrow M^0 L^0 T^0 = \left[\frac{M}{LT}\right]\left[L\right]^a\left[\frac{L}{T}\right]^b\left[\frac{M}{L^3}\right]^c$

M: $1 + c = 0 \Rightarrow c = -1$
T: $-1 - b = 0 \Rightarrow b = -1$
L: $-1 + a + b - 3c \Rightarrow -1 = a$

$\rightarrow \boxed{\Pi_3 = \frac{\mu}{\rho V d}}$ (2)

$\Pi_4 = W d^a V^b \rho^c \rightarrow M^0 L^0 T^0 = \left[\frac{ML}{T^2}\right]\left[L\right]^a\left[\frac{L}{T}\right]^b\left[\frac{M}{L^3}\right]^c$

M: $1 + c = 0 \Rightarrow c = -1$
T: $-2 - b = 0 \Rightarrow b = -2$
L: $1 + a + b - 3c = 0 \Rightarrow a = -2$

$\rightarrow \boxed{\Pi_4 = \frac{W}{\rho V^2 d^2}}$

PROBLEM 1 - p.2

■ USING $\boxed{d, V, \mu}$ AS REPEATING PARAMETERS

$$\boxed{\Pi_1 = \frac{h}{d}, \quad \Pi_2 = \frac{D}{d}}$$ AS BEFORE

2

$$\Pi_3 = \rho d^a V^b \mu^c \rightarrow M^0 L^0 T^0 = \left[\frac{M}{L^3}\right]\left[L\right]^a\left[\frac{L}{T}\right]^b\left[\frac{M}{LT}\right]^c$$

$$\left.\begin{aligned} M&: 1 + c = 0 \Rightarrow c = -1 \\ T&: -b - c = 0 \Rightarrow b = 1 \\ L&: -3 + a + b - c = 0 \Rightarrow 1 = a \end{aligned}\right] \rightarrow \boxed{\Pi_3 = \frac{\rho V d}{\mu}}$$

(COULD HAVE ANTICIPATED THIS)

$$\Pi_4 = W d^a V^b \mu^c \rightarrow M^0 L^0 T^0 = \left[\frac{ML}{T^2}\right]\left[L\right]^a\left[\frac{L}{T}\right]^b\left[\frac{M}{LT}\right]^c$$

$$\left.\begin{aligned} M&: 1 + c = 0 \Rightarrow c = -1 \\ T&: -2 - b - c = 0 \Rightarrow -1 = b \\ L&: 1 + a + b - c = 0 \Rightarrow a = -1 \end{aligned}\right] \rightarrow \boxed{\Pi_4 = \frac{W}{\mu V d}}$$

■ USING d, V, ρ: GET $\Pi_1 = fn(\Pi_2, \Pi_3, \Pi_4)$

$$\Rightarrow \frac{h}{d} = fn\left(\frac{D}{d}, \frac{\mu}{\rho V d}, \frac{W}{\rho V^2 d^2}\right) \quad (*)$$ 1

USING $d, V, \mu \Rightarrow \frac{h}{d} = fn\left(\frac{D}{d}, \frac{\rho V d}{\mu}, \frac{W}{\mu V d}\right) \quad (**)$ 1

• NOTE: $\frac{\rho V d}{\mu} = \left(\frac{\mu}{\rho V d}\right)^{-1}$ (OBVIOUSLY); AND $\frac{W}{\mu V d} = \frac{W}{\rho V^2 d^2} \cdot \frac{\rho V d}{\mu}$ 2

SO (**) CAN BE WRITTEN $\frac{h}{d} = fn\left(\frac{D}{d}, \left(\frac{\mu}{\rho V d}\right)^{-1}, \left(\frac{W}{\rho V^2 d^2}\right)\left(\frac{\mu}{\rho V d}\right)^{-1}\right)$

e.g. IF YOU CAN WRITE $Y = fn(x)$, YOU CAN ALSO WRITE $Y = fn(1/x)$

WHICH IS EQUIVALENT TO $= fn\left(\frac{D}{d}, \frac{\mu}{\rho V d}, \frac{W}{\rho V^2 d^2}\right)$ SINCE ALL WE KNOW ABOUT 'fn' IS WHICH PARAMETERS ARE INVOLVED...

THUS, (*) AND (**) ARE EQUIVALENT AS FAR AS ENUMERATING THE RELEVANT PARAMETERS.

PROBLEM 2

a

$$V_T = fn(\omega, \rho, \mu, D) \rightarrow n=5,\ m=3,\ n-m=2 \text{ DIMENSIONLESS PARAMETERS}$$ (1)

$V_T \rightarrow \left[\frac{L}{T}\right]$, $\omega \rightarrow \left[\frac{1}{T}\right]$, $\rho \rightarrow [M/L^3]$, $\mu \rightarrow \left[\frac{M}{LT}\right]$, $D \rightarrow [L]$

USE ρ, μ, D AS REPEATING PARAMETERS

$$\Pi_1 = V_T \cdot \rho^a \mu^b D^c \rightarrow M^0 L^0 T^0 = \left[\frac{L}{T}\right]\left[\frac{M}{L^3}\right]^a\left[\frac{M}{LT}\right]^b\left[L\right]^c$$

FOR
- T: $-1-b=0 \Rightarrow b=-1$
- M: $a+b=0 \Rightarrow a=1$
- L: $1-3a-b+c \overset{=0}{\Rightarrow} c=1$

$$\boxed{\Pi_1 = \frac{V_T \rho D}{\mu}}$$

$$\Pi_2 = \omega \rho^a \mu^b D^c \rightarrow M^0 L^0 T^0 = \left[\frac{1}{T}\right]\left[\frac{M}{L^3}\right]^a\left[\frac{M}{LT}\right]^b\left[L\right]^c$$ (3)

- T: $-1-b=0 \Rightarrow b=-1$
- M: $a+b=0 \Rightarrow a=1$
- L: $-3a-b+c=0 \Rightarrow c=2$

$$\boxed{\Pi_2 = \frac{\omega \rho D^2}{\mu}}$$

b IF μ ISN'T IMPORTANT: $V_T = fn(\omega, \rho, D)$ $\quad n=4,\ m=3 \rightarrow$ ONE Π PARAMETER (1)

- REPEATING PARAMETERS HAVE TO BE ω, ρ, D

$$\Pi_1 = V_T\, \omega^a \rho^b D^c \rightarrow M^0 L^0 T^0 = \left[\frac{L}{T}\right]\left[\frac{1}{T}\right]^a\left[\frac{M}{L^3}\right]^b\left[L\right]^c$$

- M: $b=0$
- T: $-1-a=0 \Rightarrow a=-1$
- L: $1-3b+c=0 \Rightarrow c=-1$

$$\boxed{\Pi_1 = \frac{V_T}{\omega D}}$$ (2)

(OVER)

PROBLEM 2 - p.2

$\boxed{c}$ IF ω ISN'T IMPORTANT: $V_T = Fn(\rho, \mu, D) \longrightarrow$ ONE Π PARAMETER (1)

ALREADY KNOW THIS FROM $\boxed{a} \longrightarrow \boxed{\Pi_1 = \frac{V_T \rho D}{\mu}}$ (2)

$\boxed{d}$ • IF μ IS NOT IMPORTANT:

RESULT FROM $\boxed{b}$ TELLS US THAT $\underset{\text{SOME FUNCTION}}{Fn}\left(\frac{V_T}{\omega D}\right) = 0 \Rightarrow \boxed{\frac{V_T}{\omega D} = \text{CONST.}}$ (1)

• IF ω IS NOT IMPORTANT:

$\boxed{c}$ TELLS US THAT $\underset{\text{SOME FUNCTION}}{Fn}\left(\frac{V_T \rho D}{\mu}\right) = 0 \Rightarrow \boxed{\frac{V_T \rho D}{\mu} = \text{CONST.}}$ (1)

• CHECK THESE FOR THE TABULATED DATA

(1)

$\frac{V_T}{\omega D}$	$\frac{V_T \rho D}{\mu}$
$1.33 \cdot 10^{-3}$	$1.50 \cdot 10^{5}$
$1.24 \cdot 10^{-2}$	$1.55 \cdot 10^{5}$
$7.03 \cdot 10^{-3}$	$1.52 \cdot 10^{5}$
$8.8 \cdot 10^{-2}$	$1.49 \cdot 10^{5}$
0.924	$1.51 \cdot 10^{5}$

THESE VARY SIGNIFICANTLY

THESE ARE NEARLY UNIFORM

(2)

$\therefore$ THE DATA SUGGEST THAT ω IS NOT IMPORTANT, SINCE THE ANALYSIS FROM $\boxed{c}$ IS CONSISTENT WITH THE DATA