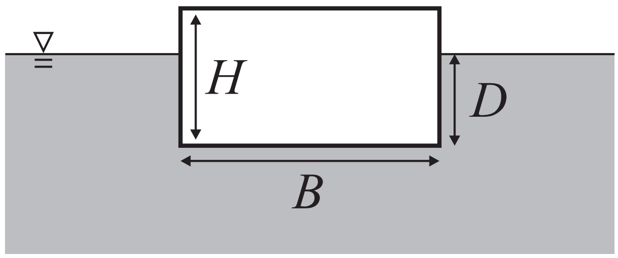
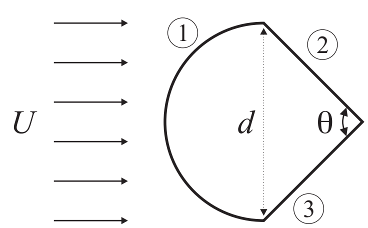
530.327 - Introduction to Fluid Mechanics - Su
HW 3 - due 30 September
60 pts

Problem 1. (10 pts) Consider a barge shaped like a rectangular slab, with height H and width (for ships, this is known as the 'beam') B, and which extends a length L in the out-of-page direction. Suppose that the barge has a uniform density, that its total mass is M, and that when placed in water, its draft (the distance from the water surface to the bottom of the barge) is D. By considering the pressure forces acting on the outside of the barge, show that the buoyant force is equal to the weight of displaced water.

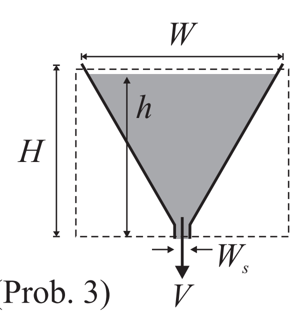


Problem 2. (10 pts) The control volume shown has three sides: side 1 is a half-circle with diameter d, while sides 2 and 3 have identical length and meet with vertex angle $\theta = 90°$. The control volume extends a length l in the out-of-page direction. Suppose you have a steady, uniform fluid flow where all fluid particles move in the same direction, in the plane of the page, with velocity U. The control volume is oriented so that side 1 faces directly upstream and the vertex of sides 2 and 3 faces straight downstream. Compute the flux term in the mass conservation equation (Eq. 4.12 in the text) for faces 1 and 2 separately by explicitly evaluating the appropriate integrals.

Problem 3. (15 pts) A long trough has an essentially triangular cross-section, as shown in the figure. The trough has height $H = 2\,\text{ft}$, spans a width at the top $W = 2.4\,\text{ft}$, and extends $L = 10\,\text{ft}$ in the out-of-page direction. At the bottom of the trough, water at 70°F flows out of a slot that is $W_s = 0.25$ inches wide. As we will learn later, the velocity of the water flow at the slot exit relates to the height of the water above the slot opening, h, as $V = \sqrt{2gh}$, where $g = 32.2\,\text{ft/s}^2$. If the initial height of water in the trough is $h = 1.8\,\text{ft}$, how long will it take for the water level to fall to $h = 1.5\,\text{ft}$? Use the control volume shown, where the top of the CV is above the free surface and the bottom of the CV is just above the slot exit; assume that the water velocity at the slot exit is uniform across the span of the slot; and, for the purpose of calculating the first integral in Eq. 4.12, you can treat the trough cross-section as a triangle that comes to a point at the bottom.



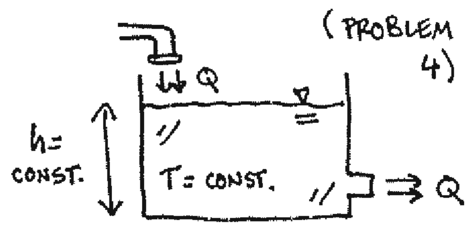
(Prob. 2)



(Prob. 3)

(over)

Problem 4. (15 pts) Anyone who's made lemonade at home knows that it requires a truly frightening amount of sugar. Suppose that you own a lemonade business that uses a recipe calling for 160 grams of sugar per liter of water. Your business makes use of the tank arrangement shown below. A sugar solution (concentration 500 grams of sugar per liter of water) flows out of a faucet at a constant volume flow rate $Q = 1\,\mathrm{L/s}$ and enters the tank. The tank contains 1000 liters of water and has a regulated valve on the side that maintains a constant liquid depth, h. The water in the tank is initially free of sugar. Assume that dissolving sugar in water doesn't change the volume of the water, that the sugar in the tank is uniformly distributed at all times, and that everything is at a uniform temperature of 20°C. How long will it take for the water in the tank to contain 160 grams of sugar per liter of water? Solve this problem using the control volume equation, Eq. 4.10, and define your control volume carefully (there is one possible definition that we know that is far superior to the other). In applying this equation, define the density, ρ, to be the density of the water only (ignoring the dissolved sugar). With that definition of ρ, explain briefly how you define N, dN/dt, and η.



Problem 5. (10 pts) The figure shows the top view of a water tank with a square cross-section that spans $L = 10\,\mathrm{ft}$ per side, and which is 6 ft tall (in the out-of-page direction). One side of the tank consists entirely of a door that is hinged along one of the vertical corners. Suppose that the tank is filled to the brim with water at 70°F. To hold the door in place, you use a jet of water (also at 70°F), with constant diameter $D = 3$ inches, that strikes the door at a distance $d = 1\,\mathrm{ft}$ from the end of the door. The jet has uniform velocity V. After hitting the door, the fluid particles all travel in a direction parallel to the door. What does V have to be in order to keep the door closed? Show your control volume clearly. (Use a value of $g = 32.2\,\mathrm{ft/s^2}$.)